CSCI 433/502 Algorithms
Self Test
Analysis of Recursion

1. Use substitution/unrolling to solve the following recurrences. Show your work.

(a) \( T(n) = T(n/2) + n \) \( T(1) = 1 \) assume \( n = 2^k \) for some integer \( k \)
(b) \( T(n) = T(n-1) + n \) \( T(0) = 0 \)
(c) \( T(n) = 4T(n/2) + 1 \) \( T(1) = 1 \) assume \( n = 2^k \) for some integer \( k \)

2. Consider the following algorithm.

Algorithm `outside-in` (int `E[]`, first, last)
if first < last
  find the positions of the max and min elements in `E[first]` ... `E[last]`
  swap `E[first]` with `E[index_of_min]`
  swap `E[last]` with `E[index_of_max]`
  `outside-in` (`E`, first+1, last-1)

(a) What does the algorithm do?
(b) Write a recurrence relation to express the complexity of the algorithm. Indicate the basic unit you are counting.
(c) Solve the recurrence relation by substitution (unrolling).

3. Celebrity problem. A celebrity among a group of \( n \) people is a person who knows nobody but is known by everybody else. The task is to identify a celebrity by only asking questions to people of the form “Do you know him/her?” Design an efficient algorithm to identify a celebrity or determine that the group has no such person. How many questions does your algorithm need in the worst case?